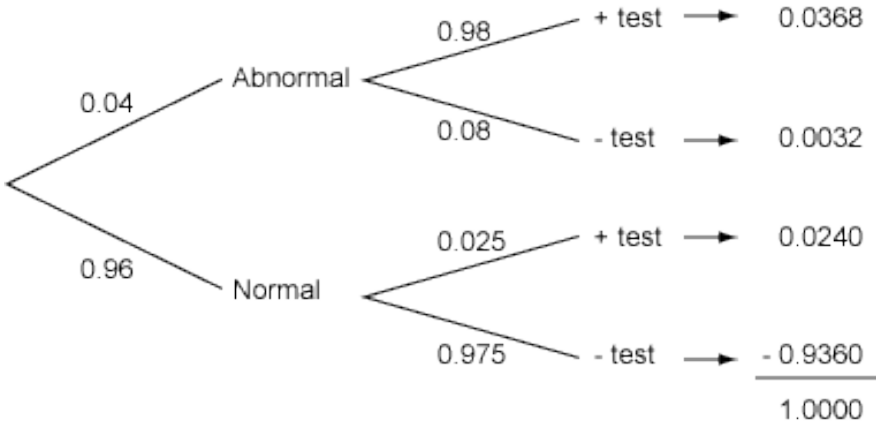
Answers: Free-Response Questions

1. **A certain genetic abnormality occurs in roughly 4% of the population. A blood test will detect the abnormality in 92% of those who have it. However, a false-positive result occurs 2.5% of the time for those who do *not* have the abnormality.**

   A. Draw a tree diagram to illustrate the situation. Include all probabilities. (6 points)

   Answer:



   Scoring notes: 6 points awarded for part (A) if tree diagram is drawn correctly with correct labeling and probabilities. Award 3 points maximum if final individual branch probabilities are not provided.

   B. What is the probability that a randomly chosen individual will test negative for the abnormality? (6 points)

   Answer: P(neg. test) = .04(.08) + .96(.975) = .0032 + .936 = .9392

   Scoring notes: 6 points awarded for part (B) only if work is done correctly.

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C. Given that a person has tested negative, what is the probability that he or she has the abnormality? (8 points)

Answer: Conditional probability: P(has abnormality given that tests negative) = P(both) / P(neg. test) = .0032/.9392 = .0034

> **Scoring notes:** 8 points awarded for part (C) for correct use of conditional probability formula. Note that if student uses a wrong answer from part (B) — but uses the correct formula — he or she should still be given full credit.

2. The weighted GPAs for 30 AP Statistics students are shown below. For convenience, the data have been ordered.

3.3	3.5	3.5	3.7	3.76	3.76	3.8	3.88	3.9	4.0
4.0	4.04	4.1	4.1	4.2	4.3	4.3	4.3	4.3	4.39
4.42	4.43	4.5	4.5	4.53	4.56	4.57	4.6	4.7	4.76

A. Explain how you would determine if there are any outliers in the data. Be specific. (8 points)

Answer: Observations below Q1 – 1.5(IQR) and observations above Q3 + 1.5(IQR) will be outliers. In this case, there are no outliers since no observations are less than 3.88 – 1.5(.62) = 2.95 or greater than 4.5 + 1.5(.62) = 5.43.

> **Scoring notes:** For full credit to be awarded, the student should demonstrate an understanding of what constitutes an outlier *and* also show the calculations using information from the five-number summary. Award 4 points if only one of the two is shown.

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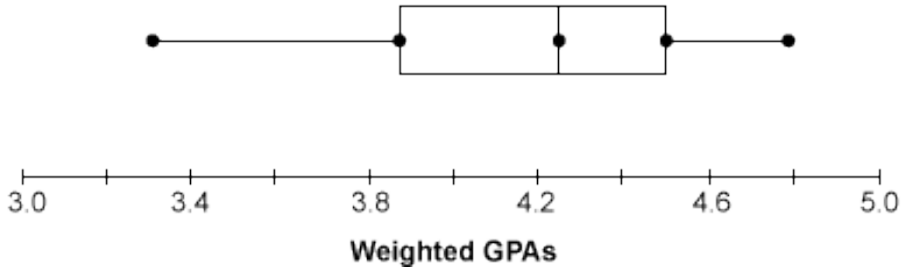
B. Describe the distribution using an appropriate graph and numerical summary. Note any unusual features. (12 points)

Answer:



Student may choose any appropriate graph. Image shows a box plot, but a stem plot, dot plot, or even a histogram could be used. Proper scaling and labeling should be used in the graph. The five-number summary for the data is as follows: min = 3.3, Q1 = 3.88, med = 4.25, Q3 = 4.5, max = 4.76. Shape of graph is skewed left with no outliers. The center is the median, which is 4.25. The spread is shown using either the range 3.3 to 4.76 or the IRQ 3.88 to 4.5 and standard deviation of .3788.

Scoring notes: 6 points awarded for a correct graph with labels. 2 points each awarded for addressing center, shape, and spread for a maximum of 6 points.

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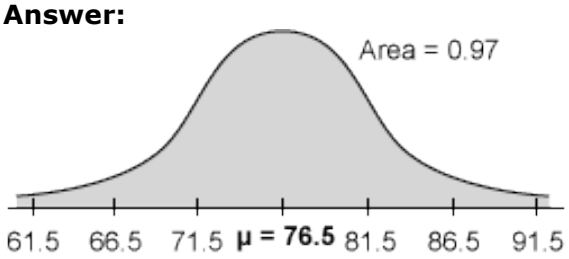
3. State end-of-grade tests tend to follow a normal distribution. In 2008, the mean test score for 8th graders was 76.5 with a standard deviation of 5. For each part of the problem below, show your work including a sketch with corresponding area shown.

A. If a student scored at the 97th percentile, what score would he or she have? (8 points)

Answer:



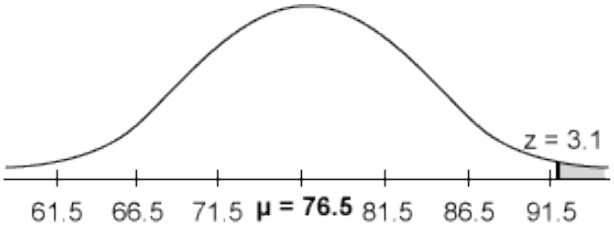
A Z-score that corresponds to an area of .97 under the normal curve is 1.88.
Using the formula x = $z\sigma + \mu$ = 1.88(5) + 76.5 = 85.9 or ≈86
InvNorm(.97,76.5,5)=85.904.

B. What percent of students score higher than 92? (6 points)

Answer:

$z = \frac{x - \mu}{\sigma} = \frac{92 - 76.5}{5} = 3.1$

The area that corresponds to z = 3.1 is 1 - .9990 = .001.
normalcdf(3.1, E99)=9.676X10^4
About 0.10% score higher than 92.



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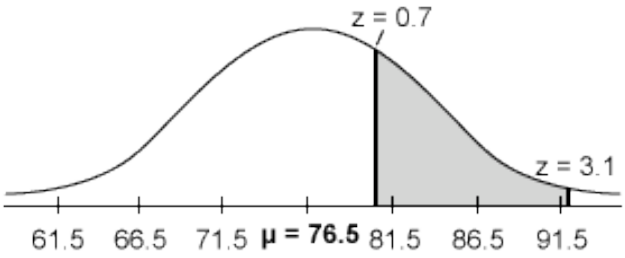
C. What percent of students score between 80 and 92? (6 points)

Answer: Z-score for x = 80 is 0.7 and z-score for x = 92 is 3.1 (from above)

Calculation is $z = \frac{80 - 76.5}{5} = 0.7$.



The area for $0.7 \leq z \leq 3.1$ = .9990 - .7580 (from table) = .2410.
.normalcdf(.7,3.1)=.2410

Scoring notes: Students may use calculator or table to find areas. They should show calculations for z-scores for all three parts of the problem and supporting diagrams with appropriate area shaded. If only an answer is shown with no supporting work, 1 point should be awarded.

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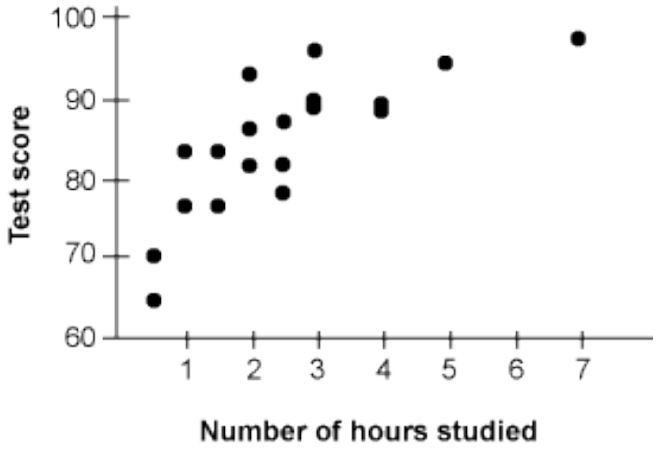
4. The scatterplot below shows the number of hours spent studying for a physics test and the corresponding test grade for 20 students.



The least-squares regression line for the scatter plot is:

$\widehat{test\ grade}$ **= 73.7788 + 4.1830 # of hours studied.**

A. Interpret the slope of the least-squares regression line. (5 points)

Answer: For each increase in 1 hour of study time, there will be, on average, a corresponding increase of about 4.2 points in the test score.

Scoring notes: 5 points will be awarded if student gives a correct interpretation of slope with appropriate use of the words, on average, or something similar such as "we can expect" or "we can predict." 2 points awarded if this is left out.

B. Given that s_x = 1.5751 and s_y = 8.6415 calculate the correlation, r. (5 points)

Answer:

Use the formula, $b = r\left(\frac{s_y}{s_x}\right)$ to get $4.1830 = r\left(\frac{8.6415}{1.5751}\right)$ so that $4.1830 = 5.4863r$
$r = .7624$

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C. Use your answer from part (B) and calculate r^2 . Interpret this value. (5 points)

Answer: r^2 = (.7624)² = .5813; about 58.13% of the variation in test scores can be accounted for by the linear relationship with the number of hours studied.

Scoring notes: 1 point awarded for correct calculation of r-squared; 4 points for a correct interpretation.

D. One student was absent on the day of the test. He studied for 6 hours and scored a 70 on the test. What effects, if any, does this point have on the least-squares line and correlation? Explain. (5 points)

Answer: This point is influential. Since it is in the extreme x-direction, it will pull the LSRL towards it. This results in a lower correlation, decreased slope, and increased y-int., basically changing the position of the LSRL.

Scoring notes: Full credit should be awarded as long as the student comments that the point is influential and will therefore change the position of the LSRL and lower the correlation. Both position of the line and correlation must be addressed for full credit to be awarded.

5. Researchers have developed a new computer-based self-paced course to teach algebra to students of varying ability. To test the effectiveness of the new method, researchers will administer a pre-test to 60 students taking part in the study. Half the students will be taught algebra in a traditional classroom, while the other half will receive instruction via the new method. A post-test will be given to all students upon completion of their instruction, and post-test results will be compared to pre-test results.

A. Is this an observational study or an experiment? (6 points)

Answer: This is an experiment because a treatment (type of instruction) is being imposed.

Scoring notes: This part is graded as either correct or incorrect.

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B. Explain how you would randomize. That is, how would you decide which children get taught with which method? (6 points)

Answer: The student must provide a correct method for randomly assigning the students to each of the two instructional methods. One possible way is to arrange the children in alphabetical order and assign them a number from 1 to 60. Then, use a random number generator to generate numbers. The first 30 numbers generated from 1 to 60, ignoring any repeats, will be taught using the new method, and the remaining 30 will be taught in the traditional classroom.

Scoring notes: Student is awarded 6 points for any valid method of randomly assigning the students to an instructional method. If a random number generator is used, it should be noted that repeated numbers need to be ignored.

C. Imagine that researchers believe that children with high math ability are more likely to excel with this program than those with low math ability. Describe a matched-pairs design that could be used to test the researchers' belief. (8 points)

Answer: Pair the students up according to their pre-test scores — the two highest scores will be a pair, and then the next two highest, and so on. Randomly assign one student in each pair to the new method and the other one to the classroom instruction. This could be done using a coin toss or some other method. After the instruction is completed, administer a post-test and compare scores for each pair.

Scoring notes: 8 points awarded for a correct design and addressing randomization. If the student has a valid design but fails to address randomization, then award 4 points. If the student's design does not meet either of these situations, then no credit is awarded.

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